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## Quantum radiation from density variations in dielectrics

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**Abstract.** Fluctuating electrostrictive forces inside a dielectric lead to the radiation of photons if the dielectric is deformed or displaced by external forces. This is a generalization of the effect of radiation from moving mirrors to physically realistic systems.

Quantum radiation from moving mirrors was discovered more than 20 years ago [1, 2], and it has since been shown that the effect is not specific to perfectly conducting mirrors but occurs in much the same way for moving dielectric mirrors [3]. However, all studies to date have regarded the mirror as a rigid body which makes the radiation appear to originate from the surface of the mirror or from the interface of two dielectric media of different polarizability. A rigid body is, of course, a wholly artificial model; in reality all material bodies are compressible. Dropping the assumption of rigidity one discovers a whole range of new effects previously ignored.

This paper centres on the investigation of the interaction of a compressible dielectric with the electromagnetic field and, in particular, with the vacuum fluctuations of the field. The question posed is whether an arbitrary change in the density of a dielectric radiates photons and what the characteristics of this radiation is. The dielectric is supposed to be non-dispersive and non-absorbing, which is always a valid assumption for a model since one can restrict oneself to a particular range of frequencies where dispersion is absent, though, for simplicity, the calculations to follow will make this assumption for all frequencies<sup>‡</sup>. Thus, the dielectric can be described by a local dielectric constant  $\epsilon(\mathbf{r}, t)$  which is a certain function of the local density  $\rho(\mathbf{r}, t)$  of the material but is supposed not otherwise to depend on any varying parameter. In particular, any temperature dependence of  $\epsilon$  is ignored for the present purposes<sup>§</sup>. The density distribution  $\rho(\mathbf{r}, t)$  is assumed to satisfy the equation of continuity

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{\beta}) = 0 \quad (1)$$

where  $\boldsymbol{\beta}(\mathbf{r}, t)$  is the local flow velocity of the material<sup>||</sup>.

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<sup>‡</sup> Dispersion cannot be ignored if the time-dependence of the density  $\rho$  (and hence of the dielectric constant  $\epsilon$ ) is such that it includes significant Fourier modes (phonons) at a frequency where  $\epsilon$  is strongly dispersive. Then the coupling of these modes to dissipation in the dielectric will dominate the physical processes in the system.

<sup>§</sup> Next-to-leading-order corrections to the effects considered here come from local compressional heating.

<sup>||</sup>  $\hbar$  and  $c$  are set equal to one throughout this paper. The velocity  $\boldsymbol{\beta}$  is always assumed to be small compared with one, i.e. any material motion is supposed to be slow compared with the speed of light.

If the dielectric is linear and non-magnetic, then the relations  $\mathbf{D}' = \epsilon \mathbf{E}'$  and  $\mathbf{B}' = \mathbf{H}'$  are valid in the individual rest-frames of every volume element of the dielectric. Transformation into the laboratory frame where the dielectric moves with the velocity  $\boldsymbol{\beta}(\mathbf{r})$  then leads to the constitutive equations

$$\mathbf{E} = \frac{1}{\epsilon} \mathbf{D} - \frac{\epsilon - 1}{\epsilon} \boldsymbol{\beta} \times \mathbf{B} \quad \mathbf{H} = \mathbf{B} + \frac{\epsilon - 1}{\epsilon} \boldsymbol{\beta} \times \mathbf{D} \quad (2)$$

where terms of order  $\beta^2$  and higher have already been discarded (cf equation (278) of [4] and equation (76.9) of [5]). The above consideration ignores effects of spatial dispersion, which, however, is innocuous in the case of dielectrics as long as the wavelengths of any density variation are longer than atomic dimensions (cf section 103 of [5], or [6]).

Proceeding from the material-independent expression for the Hamiltonian density for the electromagnetic fields

$$\mathcal{H} = \frac{1}{2} (\mathbf{D} \cdot \mathbf{E} + \mathbf{B} \cdot \mathbf{H}) \quad (3)$$

one can then easily construct the Hamiltonian for the fields in the presence of a time-varying dielectric by expressing the  $\mathbf{E}$  and  $\mathbf{H}$  fields in equation (3) through the constitutive relations (2). This gives the Hamiltonian  $H = H_0 + \Delta H$  with the unperturbed part

$$H_0 = \frac{1}{2} \int_{\mathbb{R}^3} d^3\mathbf{r} \left( \frac{\mathbf{D}^2}{\epsilon[\rho(\mathbf{r}, t)]} + \mathbf{B}^2 \right) \quad (4)$$

and the perturbation

$$\Delta H = \int_{\mathbb{R}^3} d^3\mathbf{r} \frac{\epsilon - 1}{\epsilon} \boldsymbol{\beta}(\mathbf{r}, t) \cdot (\mathbf{D} \times \mathbf{B}). \quad (5)$$

This is a very natural result. The first term, (4), is just the zero-order Hamiltonian for the medium at rest but with a parametric dependence on the (local) dielectric constant and hence on the particular density distribution at any given moment. The second term, (5), is the energy flux of the moving polarization field that necessarily accompanies any material motion  $\boldsymbol{\beta}(\mathbf{r}, t)$ .

Alternatively one can derive the above Hamiltonian from first principles by way of a Lagrangian (see appendix A of [7] for that). It turns out that the Lagrangian density for a moving dielectric is uniquely determined by three basic requirements: (i) that it is a Lorentz scalar, (ii) that in the limit  $\epsilon = 1$  it reduces to the Lagrangian density for the fields in vacuum, and (iii) that in the limit  $\boldsymbol{\beta} = 0$  it turns into the Lagrangian density for the fields in a stationary dielectric.

The system is quantized by diagonalizing the zero-order Hamiltonian  $H_0$  of equation (4), i.e. by mapping it into the Hamiltonian for a bath of harmonic oscillators that represent photons of wavevector  $\mathbf{k}$  and polarization  $\sigma$ ,

$$H_0 = \sum_{\sigma} \int d^3\mathbf{k} \omega (a_{\mathbf{k},\sigma}^{\dagger} a_{\mathbf{k},\sigma} + \frac{1}{2}). \quad (6)$$

This is achieved by the standard mode expansion

$$\begin{aligned} \mathbf{D}(\mathbf{r}) &= \epsilon(\mathbf{r}) \sum_{\sigma} \int d^3\mathbf{k} \frac{i\omega}{\sqrt{\omega}} (a_{\mathbf{k},\sigma} \mathbf{A}_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma) - \text{h.c.}) \\ \mathbf{B}(\mathbf{r}) &= \sum_{\sigma} \int d^3\mathbf{k} \frac{1}{\sqrt{\omega}} (a_{\mathbf{k},\sigma} \nabla \times \mathbf{A}_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma) + \text{h.c.}) \end{aligned}$$

with mode functions that satisfy the generalized Coulomb gauge condition  $\nabla \cdot \{\epsilon(\mathbf{r}) \mathbf{A}_{[\rho]}(\mathbf{r})\} = 0$  and are solutions of the wave equation

$$\omega^2 \epsilon[\rho] \mathbf{A}_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma) - \nabla \times \nabla \times \mathbf{A}_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma) = 0. \quad (7)$$

This equation is the eigenvalue equation of the Hermitean operator  $1/\sqrt{\epsilon}(\nabla \times \nabla \times)1/\sqrt{\epsilon}$ : each eigenvalue,  $\omega^2$ , belongs to a whole set of degenerate states which are labelled by a wavevector  $\mathbf{k}$  and a polarization index  $\sigma$ , as usual, and the functions  $\sqrt{\epsilon[\rho]}A_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma)$  form an orthogonal and complete set of eigenfunctions which spans the Hilbert space.

Although this eigenvalue problem is well posed, in practice it may be rather difficult to solve equation (7) exactly for all possible distributions  $\rho(\mathbf{r})$ . However, in most applications WKB theory can be resorted to for finding an approximate solution of sufficient accuracy.

The time-evolution of the state of the electromagnetic field is described by the Schrödinger equation

$$i \frac{d}{dt} |\psi(t)\rangle = (H_0 + \Delta H) |\psi(t)\rangle \tag{8}$$

with the initial condition  $|\psi(t_0)\rangle = |0\rangle$  that the field is in the ground state where no photons are present in any of the modes. The solution of this partial differential equation is somewhat more complicated than usual. Even though  $\Delta H$  is a small perturbation since the velocity  $\beta$  is very small compared with the speed of light, standard perturbation theory cannot be applied because the zero-order Hamiltonian,  $H_0$ , is a functional of the density distribution,  $\rho(\mathbf{r}, t)$ , and hence depends parametrically on time. The problem can neither be solved by straightforward application of the method of adiabatic approximation [8], but nevertheless the spirit of the adiabatic approximation can be followed in the solution. The idea is to determine the complete sets of eigenstates of the zero-order Hamiltonian  $H_0[\rho(t)]$  for all  $t$ , i.e. for every possible density distribution, and to expand the wavefunction  $|\psi(t)\rangle$  in terms of the appropriate set at time  $t$ . Inserting such an expansion into the Schrödinger equation (8) one obtains an infinite system of interdependent differential equations for the expansion coefficients. One then commonly extracts a phase from these expansion coefficients (which are the transition matrix elements of  $|\psi(t)\rangle$ ) into the momentary eigenstates  $|m\rangle$  by defining

$$\langle m | \psi \rangle = c_m \exp \left[ -i \int_{t_0}^t d\tau E_m(\tau) \right] \tag{9}$$

where  $E_m(t)$  is the parametrically time-dependent energy eigenvalue belonging to the eigenstate  $|m\rangle$ . In the following it will be assumed that the photon energies and hence these eigenvalues,  $E_m$ , are in fact independent of  $\rho(\mathbf{r}, t)$  and therefore of time. This is a valid assumption as long as the photon modes do not contain strong multiple scattering off the dielectric. (A high-Q cavity of variable length, for instance, cannot be considered in this approximation because the energies of the photon modes inside it depend on its length.) The system of differential equations for the expansion coefficients can then be solved perturbatively. The details of this modified adiabatic approximation have been reported at the beginning of section III of [7], so that the first-order solution for evolution from ground state is only quoted here. To first order in the perturbation parameter,  $\beta$ , the vacuum receives only admixtures of two-photon states  $|1_{\mathbf{k}}, 1_{\mathbf{k}'}\rangle = |\mathbf{k}, \mathbf{k}'\rangle$ , and up to a phase  $\exp[-i(\omega + \omega')(t - t_0)]$ , as indicated in equation (9), the time derivative of the transition matrix element  $\langle \mathbf{k}, \mathbf{k}' | \psi \rangle$  is given by

$$\frac{\partial c_{\mathbf{k}\mathbf{k}'}}{\partial t} = \left( \frac{1}{\omega + \omega'} \langle \mathbf{k}, \mathbf{k}' | \frac{\partial H_0}{\partial t} | 0 \rangle - i \langle \mathbf{k}, \mathbf{k}' | \Delta H | 0 \rangle \right) e^{i(\omega + \omega')(t - t_0)}. \tag{10}$$

Note that the two terms in the parentheses are of the same order. By way of the continuity equation (1) the time derivative of  $H_0$  leads to a linear dependence of this term on the velocity  $\beta$ , and the perturbation  $\Delta H$  in the other term is inherently of order  $\beta$ . This shows in retrospect that both terms are equally important, and that neither standard perturbation theory dealing solely with  $\Delta H$  nor an adiabatic treatment of  $H_0[\rho(t)]$  alone would have given correct results.

The derivative of  $H_0$  with respect to the parameter-dependence on time is given through the functional derivative with respect to the density profile  $\rho(\mathbf{r}, t)$

$$\frac{\partial H_0}{\partial t} = \int d^3\mathbf{r} \frac{\delta H_0}{\delta \rho} \frac{\partial \rho}{\partial t} \quad (11)$$

and hence one obtains

$$\frac{\partial H_0}{\partial t} = -\frac{1}{2} \int d^3\mathbf{r} \frac{D^2}{\epsilon^2} \frac{\partial \epsilon}{\partial \rho} \frac{\partial \rho}{\partial t}. \quad (12)$$

According to equation (5) one has

$$\langle \mathbf{k}, \mathbf{k}' | \Delta H | 0 \rangle = \int d^3\mathbf{r} \beta(\mathbf{r}, t) \cdot \frac{\epsilon - 1}{\epsilon} \langle \mathbf{k}, \mathbf{k}' | \mathbf{D} \times \mathbf{B} | 0 \rangle. \quad (13)$$

Since the two-photon state  $|\mathbf{k}, \mathbf{k}'\rangle$  is an eigenstate of  $H_0$  with eigenvalue  $\omega + \omega'$ , one can recast the matrix element

$$\langle \mathbf{k}, \mathbf{k}' | \mathbf{D} \times \mathbf{B} | 0 \rangle = \frac{-i}{\omega + \omega'} \langle \mathbf{k}, \mathbf{k}' | \frac{\partial}{\partial t} (\mathbf{D} \times \mathbf{B}) | 0 \rangle. \quad (14)$$

This is, of course, a zero-order identity, which however is fully sufficient at this stage of a first-order calculation. Using the continuity equation (1) and integrating by parts one can finally write for the time derivative of the transition amplitude:

$$\frac{\partial c_{\mathbf{k}\mathbf{k}'}}{\partial t} = -\frac{1}{\omega + \omega'} e^{i(\omega + \omega')(t-t_0)} \int d^3\mathbf{r} \beta(\mathbf{r}, t) \cdot \langle \mathbf{k}, \mathbf{k}' | \mathbf{f}(\mathbf{r}) | 0 \rangle \quad (15)$$

where the newly introduced quantity  $\mathbf{f}(\mathbf{r})$  reads

$$\mathbf{f} = \frac{1}{2} \rho \nabla \frac{\partial \epsilon}{\partial \rho} \frac{D^2}{\epsilon^2} + \frac{\epsilon - 1}{\epsilon} \frac{\partial}{\partial t} (\mathbf{D} \times \mathbf{B}). \quad (16)$$

Inspection reveals that this quantity is in fact the force density exerted on a compressible dielectric by time-dependent electromagnetic fields (cf formula (75.18) of [5]), whose terms are going to be analysed in more detail. In connection with equation (15) it should be emphasized that although the matrix element  $\langle \mathbf{k}, \mathbf{k}' | \mathbf{f}(\mathbf{r}) | 0 \rangle$  does not depend explicitly on time, it does so implicitly via the parameter dependence of the mode-functions  $A_{[\rho]}(\mathbf{r}; \mathbf{k}, \sigma)$  which are solutions of the Helmholtz equation (7) for a given density distribution  $\rho(\mathbf{r}, t)$  which depends on time.

The first term in the expression for the force density (16) is the force density for static fields

$$\mathbf{f}_S(\mathbf{r}) = \frac{1}{2} \rho \nabla \left( \frac{\partial \epsilon}{\partial \rho} \frac{D^2}{\epsilon^2} \right). \quad (17)$$

Integration by parts brings this into the more familiar form

$$\mathbf{f}_S(\mathbf{r}) = \frac{1}{2} \nabla \left( \rho \frac{\partial \epsilon}{\partial \rho} \frac{D^2}{\epsilon^2} \right) - \frac{1}{2} \frac{D^2}{\epsilon^2} \nabla \epsilon \quad (18)$$

which is found in all common textbooks that deal with electrostriction; the above expression is identical to formula (45) on p 145 (section 2.22) of [9] and to formula (15.12) on p 62 of [5].

The second term in the force density equation (16) comes about only for time-dependent fields. It is the rate of change of the momentum density resident in the polarization field<sup>†</sup>. By

<sup>†</sup> The question of the momentum density of the electromagnetic field in a dielectric is still marked by the afterglow of the Minkowski–Abraham controversy. This author trusts the very thorough analysis of the problem by Nelson [17] who shows that momentum rather than pseudomomentum conservation leads to  $\mathbf{E} \times \mathbf{B}$  for the momentum density of an electromagnetic field in a material medium. The interpretation of  $\Delta H$  as the energy flux of the polarization field and of the second term in equation (16) as the rate of change in the momentum density of the polarization field is therefore consistent with Nelson's conclusion. However, in practice the question is not of much relevance here because the Minkowski and the Abraham formulations differ merely by a factor of  $\epsilon$  which is negligible for weak dielectrics and  $\mathbf{B}$  and  $\mathbf{H}$  are the same in non-magnetic materials.

exploiting Maxwell's equations, applying vector identities, and integrating by parts one can also transfigure this second term. In total one can reformulate equation (16):

$$\begin{aligned}
 \mathbf{f}(\mathbf{r}) = & \frac{1}{2} \nabla \left[ \frac{D^2}{\epsilon^2} \rho \frac{\partial \epsilon}{\partial \rho} - (\epsilon - 1) \frac{D^2}{\epsilon^2} - \frac{\epsilon - 1}{\epsilon} B^2 \right] + \nabla_i \left[ \frac{\epsilon - 1}{\epsilon} \left( B_i B + D_i \frac{D}{\epsilon} \right) \right] \\
 & - \frac{1}{2} B^2 \nabla \frac{1}{\epsilon} + B(B \cdot \nabla) \frac{1}{\epsilon} + \frac{D}{\epsilon} (D \cdot \nabla) \frac{1}{\epsilon}.
 \end{aligned} \tag{19}$$

This expression may look much more cumbersome than the one in equation (16) but it is much more convenient in calculations. Neither the first term, which is a gradient, nor the second, which is a divergence, contribute to the total force acting on the dielectric as a whole. Just like the first term in the static force-density (18) they are describing only local electrostrictive compression in the dielectric and cancel out from the total force. The three terms in the second line have the same meaning attached to them as the second term in equation (18); they come from differences of the electromagnetic pressure across the dielectric and their prime contribution comes from the interfaces of the dielectric with the vacuum. If one considers just a rigid body then these are the only terms that arise, and the force on the body is then simply given by the discontinuity of the stress tensor across the surfaces.

The main statement of equation (15) and of this paper is that fluctuations of the force density cause emission of photon pairs if there is any variation in the local density so that the local material flow  $\beta(\mathbf{r}, t)$  is non-zero. These must be density variations that are driven by an external agent. If there is no external driving pressure the force density still fluctuates with a mean square of

$$\langle 0 | \mathbf{f}^2 | 0 \rangle - \langle 0 | \mathbf{f} | 0 \rangle^2 = \frac{1}{2} \int d\mathbf{k} \int d\mathbf{k}' |\langle \mathbf{k}, \mathbf{k}' | \mathbf{f} | 0 \rangle|^2 \tag{20}$$

but these fluctuations are in equilibrium and balance each other so that there is no photon emission. This equilibrium is disturbed by an external driving pressure and dissipation sets in. The microscopic mechanism of this dissipation is the radiation of photons.

This effect is not new if the dielectric moves as a whole. Then the approximation of a rigid body captures the essential physics of the problem and one is led back to phenomenon of quantum radiation by (rigid) dielectric mirrors which has been investigated for a one-dimensional scalar model in [3] and for the full Maxwell field in three dimensions in [7].

What is new is that quantum radiation arises even if the dielectric is only compressed internally. Then the fluctuating electrostrictive forces are shaken out of balance which, by virtue of equation (15), causes internal dissipation by means of photon radiation. In reality this effect occurs much more readily than the motion of a dielectric body as a whole. One of the simplest practical cases would be an oscillatory density variation, in other words, a phonon. However, Fermi's golden rule demands that the frequency of the phonon lies in the same range as the frequency of the expected photons, which makes the effect hard to realise in practice. Of course one could specifically look for it, and experiments that have been suggested to look for microwave radiation from moving mirrors may well see radiation originating from compressions inside the 'mirrors', as described above, rather than genuine Unruh, i.e. mirror-induced, radiation, or at best a combination of the two, as anticipated in [10].

Furthermore, one could ask what kind of density compressions away from harmonic oscillations can reasonably cause a dielectric fluid to radiate photons. Any non-periodic process can be analysed by decomposition into Fourier components just as well, which makes one realise that optical photons are produced only if there are sufficient high-frequency components found in the Fourier spectrum of the variation in the density of the dielectric. In other words, one can expect to see such radiation only if there is some very fast process in the system. It should be pointed out explicitly that the fastness or slowness of a process is not a statement

about velocities but about short timescales and high frequencies. An example for such a process would be a collision time which depends primarily on the strength of the interaction of the two objects colliding. It seems not unreasonable that shock waves and their collisions with boundaries are what is causing the thermal triboluminescence which has been observed in alkali halides [11, 12]. Crystals subjected to cutting, milling, or scratching in ultra-high vacuum are observed to emit short pulses of very weak light with a continuous and featureless spectrum. So far the process has not found a satisfactory explanation. Attempts to explain the effect by the decay of self-trapped excitons [13] were unconvincing at the time and have since been rebutted by advances in knowledge about the spectra of such exciton decay [14] which are very different from the spectra observed in thermal triboluminescence [12]. Therefore, it may be useful to examine the possible origin of thermal triboluminescence from shock waves in solids through the mechanism described in this paper more closely.

Further applications of this new mechanism can be envisaged, but most of them would relate to solid dielectrics. Since for simplicity this paper has considered only a continuum model for the dielectric, which is strictly only applicable to dielectric fluids, the next step is to extend the investigations to solids on the basis of a formalism, for instance, as introduced by Nelson [15].

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